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Branes with Back-ground Fields in Boundary State Formalism

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Abstract

Interaction of branes in presence of internal gauge fields is considered by using the boundary state formalism . This approach enables us to consider the problems that are not easily accessible to the canonical approach via open strings . The effects of compactification of some of the dimensions on tori are also discussed . Also we study the massless state contribution on this interaction .

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1 Introduction

D_p-branes with non zero back ground internal gauge fields, or equivalently D-branes in $B_{\mu\nu}$ fields and $U(1)$ gauge fields A_α have shown several interesting properties [1, 2, 3, 4, 5, 6, 7, 8]. On the one hand they describe bound states of D_p and D_{p-2}-branes [4] or D_p-branes and fundamental strings [3] and on the other hand they give rise to SYM theories on a non commutative space [9] . Also the study of the physics of (n,1) bound states [5, 6], creation of fundamental string between D_4-branes [7], loop corrections to superstring equations of motion [8], scattering of closed strings from D-branes [1] and dynamics and interactions of D-brane solitons [2] are another applications of D_p-branes in non zero back ground gauge fields . A useful tool for describing D_p-branes and their interactions (specially in non zero back ground gauge fields) is the boundary state formalism [1, 10, 11, 12, 13, 14, 15] . Simply the overlap of boundary states through the closed string propagator give us the amplitude of interaction of branes . By introducing back ground fields $B_{\mu\nu}$ and a $U(1)$ gauge field A_α (which lives in D-brane) to the string σ-model action one obtains mixed boundary conditions for the strings emitted by the branes . The method of boundary states enables us to study the interaction of branes with different dimensions and different internal fields some of which are not easily accessible by canonical formalism [4].

We shall also consider compactification of certain dimensions on tori . The states emitted from the branes which are wrapped around compact directions with internal back ground fields turn out to be dominantly along a certain direction not perpendicular to the brane which specified by its windings . These windings, around the compact directions are correlated with their momenta along the brane . This is in contrast to the case of pure D-brane where the closed strings only have momentum perpendicular to the brane .

In section 2 we write and solve the equations of boundary states . In section 3 we use the result of section 2 to calculate the interaction of two branes of arbitrary dimensions p_1 and p_2 with different internal $\mathcal{F}$ fields. We also find the result when part of the space is compactified on a torus. We shall also show that our result reduces to that of the known cases such as zero $\mathcal{F}$ fields, and non-compact spacetime . Finally the contribution of the massless states on the amplitude is extracted and thus interaction strength is be obtained .

Since compactification effects on the interaction of the branes do not depend on the fermions, we will consider only the bosonic string . In this article we denote a brane in the back-ground of internal fields by “m_p -brane”, *i.e.* a “mixed brane” with dimension “p”.

2 Boundary state

We begin with a σ-model action containing $B_{\mu\nu}$ field and two boundary terms [16] corresponding to the two m_{p_1} and m_{p_2}-branes gauge fields .

$$
\begin{aligned}
S = -\frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma\Big(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu} + \epsilon^{ab}B_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}\Big) \\
-\frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_1} d\sigma A^{(1)}_{\alpha_1}\partial_\sigma X^{\alpha_1} + \frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_2} d\sigma A^{(2)}_{\alpha_2}\partial_\sigma X^{\alpha_2}
\end{aligned}
\tag{1}
$$

where Σ is the world sheet of the closed string exchanged between the branes . $(\partial\Sigma)_1$ and $(\partial\Sigma)_2$ are two boundaries of this world sheet, which are at $\tau = 0$ and $\tau = \tau_0$ respectively . $A^{(1)}_{\alpha_1}$ and $A^{(2)}_{\alpha_2}$ are $U(1)$ gauge fields that live in m_{p_1} and m_{p_2}-branes . The sets $\{\alpha_1\}$ and $\{\alpha_2\}$ specify the directions on the m_{p_1} and m_{p_2}-world branes . $G_{\mu\nu}$ and $B_{\mu\nu}$ are usual back-ground fields . Vanishing the variation of this action with respect to $X^{\mu}(\sigma,\tau)$ gives the equation of motion of $X^{\mu}(\sigma,\tau)$ and boundary state equations.

Taking the $B_{\mu\nu}(X)$ and $G_{\mu\nu}(X)$ to be constant fields, thus the boundary states equations become

$$
\begin{aligned}
&\Big(\partial_\tau X^{\alpha_1} + \mathcal{F}^{\alpha_1}_{(1)\ \beta_1}\partial_\sigma X^{\beta_1} + B^{\alpha_1}{}_{i_1}\partial_\sigma X^{i_1}\Big)_{\tau=0}|B^1_x\rangle = 0 \\
&(\delta X^{i_1})_{\tau=0}|B^1_x\rangle = 0 \\
&\Big(\partial_\tau X^{\alpha_2} + \mathcal{F}^{\alpha_2}_{(2)\ \beta_2}\partial_\sigma X^{\beta_2} + B^{\alpha_2}{}_{i_2}\partial_\sigma X^{i_2}\Big)_{\tau=\tau_0}|B^2_x\rangle = 0 \\
&(\delta X^{i_2})_{\tau=\tau_0}|B^2_x\rangle = 0
\end{aligned}
\tag{2}
$$

where $\{i_1\}$ and $\{i_2\}$ show the directions perpendicular to m_{p_1} and m_{p_2}-world branes and total "field strengths" are

$$
\begin{aligned}
\mathcal{F}_{(1)\alpha_1\beta_1} &= B_{\alpha_1\beta_1} - A^{(1)}_{[\alpha_1,\beta_1]} \\
\mathcal{F}_{(2)\alpha_2\beta_2} &= B_{\alpha_2\beta_2} - A^{(2)}_{[\alpha_2,\beta_2]}.
\end{aligned}
\tag{3}
$$

Furthermore if we take m_{p_1} and m_{p_2}-branes to be at positions $\{y_1^{i_1}\}$ and $\{y_2^{i_2}\}$ respectively, we have to impose

$$
\begin{aligned}
&[X^{i_1}(\sigma,\tau) - y_1^{i_1}]_{\tau=0}|B^1_x\rangle = 0 \\
&[X^{i_2}(\sigma,\tau) - y_2^{i_2}]_{\tau_0}|B^2_x\rangle = 0
\end{aligned}
\tag{4}
$$

The solution to the equation of motion is

$$
X^{\mu}(\sigma,\tau) = x^{\mu} + 2\alpha' p^{\mu}\tau + 2L^{\mu}\sigma + \frac{i}{2}\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}(\alpha^{\mu}_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}^{\mu}_m e^{-2im(\tau+\sigma)}) \tag{5}
$$

where L^μ is zero for non-compact directions but can be non-zero for the components along the compact directions, in which

$$L^\mu = N^\mu R^\mu \quad , \quad (N^\mu \in Z) \quad (no\ sum\ on\ \mu) \tag{6}$$

where R^μ is the radius of compactification in the compact direction X^μ . Also for the momenta in such directions we have

$$p^\mu = \frac{M^\mu}{R^\mu} \quad , \quad (M^\mu \in Z) \tag{7}$$

N^μ is the winding number and M^μ is the momentum number of the closed string state . Imposing the boundary conditions on the solution (5) as operator equation, we obtain the boundary state equations that, for the second brane take the form

$$\left(p^{\alpha_2} + \frac{1}{\alpha'}\mathcal{F}_{(2)}{}^{\alpha_2}{}_{\beta_2} L^{\beta_2}\right) \mid B_x^2, \tau_0\rangle = 0 \tag{8}$$

$$\left((1-\mathcal{F}_2)^{\alpha_2}{}_{\beta_2}\alpha_n^{\beta_2} e^{-2in\tau_0} + (1+\mathcal{F}_2)^{\alpha_2}{}_{\beta_2}\tilde{\alpha}_{-n}^{\beta_2} e^{2in\tau_0}\right) \mid B_x^2, \tau_0\rangle = 0 \tag{9}$$

$$\left(\alpha_n^{i_2} e^{-2in\tau_0} - \tilde{\alpha}_{-n}^{i_2} e^{2in\tau_0}\right) \mid B_x^2, \tau_0\rangle = 0 \tag{10}$$

$$(x^{i_2} + 2\alpha' p^{i_2}\tau_0 - y_2^{i_2})|B_x^2, \tau_0\rangle = 0 \tag{11}$$

$$L^{i_2} \mid B_x^2, \tau_0\rangle = 0 \tag{12}$$

where τ_0 is the τ variable on the boundary of the closed string world sheet. If the direction X^{i_2} is non-compact we have $L^{i_2} = 0$, and if this direction is compact then equation (12) implies that $L^{i_2} = 0$, *i.e.* the closed string cannot wind around directions perpendicular to the brane . Equation (8) also implies that the closed string can have a momentum in the world brane directions . This is in contrast to the non compact case where the momentum components along the brane directions are zero . As equation (8) shows the momentum and the windings of the emitted string are proportional . Physically it means that when the back ground $\mathcal{F}$ is turned on and brane is wrapped around the compact directions, the closed string that is emitted (absorbed) from (by) the brane must have its momentum in one direction of brane proportional to its winding numbers in the other compact directions of brane . According to this equation, closed string momentum components along both compact and non compact directions of the brane is quantized. It is rather surprising that the internal momentum even in non compact directions of the brane is non zero and quantized . It is worth noting that although this momentum is non zero, it is not an independent quantum number. This is the effect of internal gauge fields and compactification. For the compact directions of brane also we have quantized momenta $p^{\alpha_c} = \frac{M^{\alpha_c}}{R^{\alpha_c}}$, therefore

$$M^{\alpha_c} = -\frac{1}{\alpha'} \sum_{\beta_c} \mathcal{F}^{\alpha_c}{}_{\beta_c} N^{\beta_c} R^{\beta_c} R^{\alpha_c} \tag{13}$$

In terms of $\phi^{\alpha_c}{}_{\beta_c} = 4\pi^2 \mathcal{F}^{\alpha_c}{}_{\beta_c} R^{\alpha_c} R^{\beta_c}$, the flux through the 2-cycle $\{\alpha_c \beta_c\}$ we have,

$$M^{\alpha_c} = -\frac{1}{4\pi^2 \alpha'} \sum_{\beta_c} \phi^{\alpha_c}{}_{\beta_c} N^{\beta_c} \tag{14}$$

As M^{α_c} and N^{β_c} are integers we can look at this relation as a constraint on the flux allowing a closed string with momentum numbers $\{M^{\alpha_c}\}$ and winding numbers $\{N^{\alpha_c}\}$. M^{α_c} and N^{β_c} being integers restrict $-\frac{1}{4\pi^2\alpha'}\phi^{\alpha_c}{}_{\beta_c}$ to be rational . On the other hand for a given flux the relation between momentum numbers and winding numbers of closed strings specifies which string lives in $\mid B\rangle$ and can be emitted. Similar effect for the open strings ending on the branes is observed [9] .

Now we solve the boundary state equations . Equation (11) and (12) have the solution

$$e^{i\alpha' \tau_0 \sum_{i_2} (p_{op}^{i_2})^2} \delta^{(d-p_2-1)}(x^{i_2} - y_2^{i_2}) \prod_{i_2} \mid p_L^{i_2} = p_R^{i_2} = 0\rangle \tag{15}$$

solution to the equation (8) is,

$$\sum_{\{p^{\alpha_2}\}} \prod_{\alpha_2} \mid p^{\alpha_2}\rangle \tag{16}$$

where

$$p^{\alpha_2} = -\frac{1}{\alpha'} \mathcal{F}_{(2)}^{\alpha_2}{}_{\beta_{2c}} \ell^{\beta_{2c}} \tag{17}$$

β_{2c} is index for compact directions of the set $\{X^{\alpha_2}\}$, and $\ell^{\beta_{2c}}$ is $N^{\beta_{2c}} R^{\beta_{2c}}$.

The solution of the oscillator parts of boundary state equations is

$$e^{-\sum_{m=1}^{\infty} \frac{1}{m} e^{4im\tau_0} \alpha_{-m}^{\mu} S_{\mu\nu}^{(2)} \tilde{\alpha}_{-m}^{\nu}} \mid 0\rangle \tag{18}$$

where the matrix $S^{\mu}_{(2)\ \nu}$ is

$$S^{\mu}_{(2)\ \nu} = (Q^{\alpha_2}_{(2)\ \beta_2} \ , \ -\delta^{i_2}{}_{j_2}) \tag{19}$$

$$Q_2 \equiv (1 - \mathcal{F}_2)^{-1}(1 + \mathcal{F}_2) \tag{20}$$

Note that since $\mathcal{F}_2$ is antisymmetric Q_2 is an orthogonal matrix . Putting all these together, we obtain the boundary state

$$\mid B_x^2, \tau_0\rangle = \sum_{\{p^{\alpha_2}\}} \mid B_x^2, \tau_0\ ,\ p^{\alpha_2}\rangle \tag{21}$$

The summation is over all possible momenta, which can equivalently be written as sum over winding numbers $\{N^{\alpha_{2c}}\}$ using the relation (17) and

$$\begin{aligned} \mid B_x^2, \tau_0, p^{\alpha_2}\rangle &= \frac{T_{p_2}}{2}\sqrt{det(1-\mathcal{F}_2)}\ e^{i\alpha'\tau_0\sum_{i_2}(p_{op}^{i_2})^2}\delta^{(d-p_2-1)}(x^{i_2}-y_2^{i_2}) \\ &\times e^{-\sum_{m=1}^{\infty}\frac{1}{m}e^{4im\tau_0}\alpha_{-m}^{\mu}S_{\mu\nu}^{(2)}\tilde{\alpha}_{-m}^{\nu}} \mid 0\rangle \prod_{i_2} \mid p_L^{i_2} = p_R^{i_2} = 0\rangle \prod_{\alpha_2} \mid p^{\alpha_2}\rangle \end{aligned} \tag{22}$$

The constant T_{p_2} is the tension of D_{p_2}-brane and is derived in [1] and [14]. The origin of the factor $\sqrt{det(1-\mathcal{F}_2)}$ is in the path integral with boundary action [8, 17, 18] . The ghost part of boundary state is independent of $\mathcal{F}_2$ and is given by the same expression as for $\mathcal{F}_2 = 0$

$$\mid B_{gh}, \tau_0\rangle = e^{\sum_{m=1}^{\infty} e^{4im\tau_0}(b_{-m}\tilde{c}_{-m}+c_{-m}\tilde{b}_{-m})} \mid Z\rangle \tag{23}$$

where $\mid Z\rangle$ is the appropriate ghost vacuum [19] which can be written as

$$\mid Z\rangle = (c_0 - \tilde{c}_0)\ |{\downarrow\downarrow}\rangle \tag{24}$$

3 Interaction between two mixed branes

Now we can calculate the overlap of the two boundary states to obtain the interaction amplitude of branes. As compactification effects do not depend on fermions and superghosts therefore we discuss only on the bosonic case . Complete boundary state is

$$\mid B\rangle = \mid B_x\rangle \mid B_{gh}\rangle \tag{25}$$

These two mixed branes simply interact via exchange of closed strings, so that the amplitude is given by

$$\mathcal{A} = \langle B_1 \mid D \mid B_2, \tau_0 = 0\rangle \tag{26}$$

where "D" is the closed string propagator [11]. The calculation is straight forward but tedious.

Here we only give the final result;

$$
\begin{aligned}
\mathcal{A} \;=\; & \frac{T_{p_1}T_{p_2}}{4(2\pi)^{d_i}}\alpha'\sqrt{det(1-\mathcal{F}_1)det(1-\mathcal{F}_2)}\int_0^\infty dt\Big\{ \\
& \times e^{4at}\prod_{n=1}^{\infty}\Big([det(1-S_1S_2^Te^{-4nt})]^{-1}(1-e^{-4nt})^2\Big) \\
& \times\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{i_n}} e^{-\frac{1}{4\alpha' t}\sum_{i_n}(y_1^{i_n}-y_2^{i_n})^2}\prod_{i_c}\Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}}\mid\frac{i\alpha' t}{\pi(R_{i_c})^2}\Big) \\
& \times\sum_{\{N^{u_c}\}}\Big[(2\pi)^{d_u}[\prod_u\delta(p_1^u,p_2^u)]\; exp[\frac{i}{\alpha'}\ell^{u_c}(\mathcal{F}^{\alpha'_1}_{(1)\ \ u_c}y_2^{\alpha'_1}-\mathcal{F}^{\alpha'_2}_{(2)\ \ u_c}y_1^{\alpha'_2})] \\
& \times exp[-\frac{t}{\alpha'}\ell^{u_c}\ell^{v_c}(\eta_{u_cv_c}+\mathcal{F}^u_{(1)\ \ u_c}\mathcal{F}_{(2)\ \ uv_c}+\mathcal{F}^{\alpha'_1}_{(1)\ \ u_c}\mathcal{F}^{\alpha'_1}_{(1)\ \ v_c}+\mathcal{F}^{\alpha'_2}_{(2)\ \ u_c}\mathcal{F}^{\alpha'_2}_{(2)\ \ v_c})]\Big]\Big\} \quad (27)
\end{aligned}
$$

where "a" is a constant, depends on spacetime dimension $a=(d-2)/24$. The notation used in the above calculation is given below

$\{i\}\equiv$ Indices for directions perpendicular to the both mixed branes .

$\{u\}\equiv$ Indices for directions along on the both mixed branes .

$\{\alpha'_1\}\equiv$ Indices for directions along on the m_{p_1} and perpendicular to the m_{p_2}-branes .

$\{\alpha'_2\}\equiv$ Indices for directions along on the m_{p_2} and perpendicular to the m_{p_1}-branes .

These sets with $\{i_1\}$ and $\{i_2\}$ have set notations as

$$
\begin{aligned}
\{i_1\} &= \{i\}\bigcup\{\alpha'_2\} \\
\{i_2\} &= \{i\}\bigcup\{\alpha'_1\} \\
\{\alpha_1\} &= \{u\}\bigcup\{\alpha'_1\} \\
\{\alpha_2\} &= \{u\}\bigcup\{\alpha'_2\} \\
\{\mu\} = \{i_1\}\bigcup\{\alpha_1\} &= \{i_2\}\bigcup\{\alpha_2\}. \quad (28)
\end{aligned}
$$

In this amplitude $p_1^u=-\frac{1}{\alpha'}\mathcal{F}^u_{(1)\ \ v_c}N^{v_c}R^{v_c}$ and $p_2^u=-\frac{1}{\alpha'}\mathcal{F}^u_{(2)\ \ v_c}N^{v_c}R^{v_c}$. Indices $\{u_c,v_c,...\}$ show those directions of $\{X^u\}$ which are compact . d_u is dimension of $\{X^u\}$ and d_i is dimension of $\{X^i\}$. Also $\{i_n\}$ is non-compact part of $\{i\}$, and $\{i_c\}$ is compact part of $\{i\}$ region . ℓ^{u_c} as previous is $N^{u_c}R^{u_c}$. This amplitude as expected is symmetric under the exchange of indices "1" and "2" (see (26)). Although in this article we are dealing only with bosonic string, it is wroth noting that similar consideration concerning the effects of compactification works for the superstring case since these effects are independent of the fermions .

We can write this amplitude in another form in which common world volume of the world branes explicitly appears . In the sum over $\{N^{u_c}\}$ one term is obtained for $N^{u_c}=0$ for all u_c, therefore $p_1^u=p_2^u=0$. Furthermore for given fields and radii of compactification there may be other sets $\{N_s^{u_c}\}$ in which equality $\mathcal{F}^u_{(1)v_c} N_s^{v_c} R^{v_c} = \mathcal{F}^u_{(2)v_c} N_s^{v_c} R^{v_c}$ holds (one possibility is that $(\mathcal{F}_1 - \mathcal{F}_2)^u{}_{v_c} R_{v_c}$ be rational), then amplitude takes the form

$$
\begin{aligned}
\mathcal{A} \;=\; & \frac{T_{p_1} T_{p_2}}{4(2\pi)^{d_i}} \alpha' V_u \sqrt{det(1-\mathcal{F}_1) det(1-\mathcal{F}_2)} \int_0^\infty dt \Big\{ \Big(\sqrt{\frac{\pi}{\alpha' t}} \Big)^{d_{in}} e^{-\frac{1}{4\alpha' t} \sum_{i_n} (y_1^{i_n} - y_2^{i_n})^2} \\
& \times e^{4at} \prod_{n=1}^\infty \Big([det(1 - S_1 S_2^T e^{-4nt})]^{-1} (1 - e^{-4nt})^2 \Big) \prod_{i_c} \Theta_3 \Big(\frac{y_1^{i_c} - y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2} \Big) \\
& \times \Big[1 + \sum_s \Big(exp[-\frac{t}{\alpha'} \ell_s^{u_c} \ell_s^{v_c} (\eta_{u_c v_c} + \mathcal{F}^u_{(1)u_c} \mathcal{F}_{(2)u v_c} + \mathcal{F}^{\alpha'_1}_{(1)u_c} \mathcal{F}^{\alpha'_1}_{(1)v_c} + \mathcal{F}^{\alpha'_2}_{(2)u_c} \mathcal{F}^{\alpha'_2}_{(2)v_c})] \\
& \times exp[\frac{i}{\alpha'} \ell_s^{u_c} (\mathcal{F}^{\alpha'_1}_{(1)u_c} y_2^{\alpha'_1} - \mathcal{F}^{\alpha'_2}_{(2)u_c} y_1^{\alpha'_2})] \Big) \Big] \Big\} \qquad (29)
\end{aligned}
$$

where V_u is the common world volume of the two mixed branes and $\ell_s^{u_c} = N_s^{u_c} R^{u_c}$. If there are no sets $\{N_s^{u_c}\}$ then the expression in the bracket is 1. Note that for parallel mixed branes with the same dimension those terms which contain α'_1 and α'_2 disappear .

The effects of compactification are in the factors Θ_3 and the last bracket which reduce to 1 for the non compact case . Therefore the amplitude for non compact spacetime is the remainder part with the change $i_n \to i$.

Specially consider two parallel mixed branes with the same dimension p in non compact spacetime with dimension d . In this case V_u is V_{p+1} . Furthermore consider $\mathcal{F}_1 = \mathcal{F}_2 \equiv \mathcal{F}$, so we have $(S_1 S_2^T)^\mu{}_\nu = \delta^\mu{}_\nu$ therefore

$$
\begin{aligned}
\mathcal{A} \;=\; & \frac{T_p^2}{4(2\pi)^{d-p-1}} \alpha' V_{p+1} det(1-\mathcal{F}) \int_0^\infty dt \Big\{ \Big(\frac{\pi}{\alpha' t} \Big)^{(d-p-1)/2} \\
& \times e^{-\frac{1}{4\alpha' t} \sum_i (y_1^i - y_2^i)^2} e^{(d-2)t/6} \prod_{n=1}^\infty (1 - e^{-4nt})^{-d+2} \Big\} \qquad (30)
\end{aligned}
$$

Then for $T_p = \frac{\sqrt{\pi}}{2^{(d-10)/4}} (4\pi^2 \alpha')^{(d-2p-4)/4}$ after a transformation $t \to \pi/2t$, equation (30) can be interpreted as the one-loop free energy of an open string whose ends are fixed on these parallel branes [20] .

The formula (27) reduces to the previous known results on the interaction of mixed branes, for example those considered in [3, 4] . In the previous methods each particular relative configuration of two branes needs to be treated individually, which may or may not yield to the canonical quantization. In contrast, the method of boundary state can be used for any two arbitrary branes. There is also another advantage in the boundary state method . In the canonical method when the magnetic part of $\mathcal{F}$ is different from zero, the space becomes non commutative . In the present method we can go around this problem and all cases can be handled in the same way . In the following we consider an example using the general formula .

Example:

Consider two parallel m_2-branes along (X^1, X^2), with non zero components of fields $\mathcal{F}_{(1)\,01} = E$ and $\mathcal{F}_{(2)\,01} = E'$. In this case non zero components of momenta along the world volume are $p_1^0 = \frac{1}{\alpha'} nER_1$ and $p_2^0 = \frac{1}{\alpha'} nE'R_1$, where n is winding number of a given closed string states around the X^1-direction . Therefore the interaction amplitude becomes

$$\begin{aligned}
\mathcal{A} \;=\; & \frac{T_2^2}{4(2\pi)^{d-3}}\alpha' V_3 \sqrt{(1-E^2)(1-E'^2)} \int_0^\infty dt \Big\{ e^{4at} \prod_{n=1}^{\infty} \Big[(1-e^{-4nt})^{4-d} \\
& \times \Big(1 - \frac{(1+E)(1-E')}{(1-E)(1+E')} e^{-4nt} \Big)^{-1} \Big(1 - \frac{(1-E)(1+E')}{(1+E)(1-E')} e^{-4nt} \Big)^{-1} \Big] \\
& \times \Big(\sqrt{\frac{\pi}{\alpha' t}} \Big)^{d_{in}} e^{-\frac{1}{4\alpha' t}\sum_{i_n}(y_1^{i_n} - y_2^{i_n})^2} \prod_{i_c} \Theta_3 \Big(\frac{y_1^{i_c} - y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2} \Big) \\
& \times \theta(E, E', t, R_1) \Theta_3(0 \mid itR_2^2/\pi\alpha') \Big\} \qquad (31)
\end{aligned}$$

where V_3 is the world volume which can be written as $(2\pi R_1)(2\pi R_2)L$ in which R_1 and R_2 are radii of compactification of X^1 and X^2 directions respectively and L is the infinite time length . The function θ is defined by infinite series

$$\theta(E, E', t, R_1) = \frac{2\pi}{L} \sum_{n=-\infty}^{\infty} \delta[nR_1(E-E')/\alpha']\; e^{-t(1-E^2)R_1^2 n^2/\alpha'} \qquad (32)$$

For the case in which $E = E'$, this becomes $\theta(E, E, t, R_1) = \Theta_3\Big(0 \mid \frac{it(1-E^2)R_1^2}{\pi\alpha'}\Big)$, and for $E \neq E'$ it is equal to 1 .

For the non compact case the effect of non zero fields $E = E'$ appear only in the modification of tension of each brane by a factor $\sqrt{1-E^2}$. On the other hand when X^1-direction is compact and $E = E'$, the back-ground fields appear not only in the tensions but also in an extra factor $\Theta_3\Big(0 \mid it(1-E^2)R_1^2/(\pi\alpha')\Big)$ in the above expression .

4 Contribution of massless states ($B_{\mu\nu}$, $G_{\mu\nu}$, ϕ) on amplitude

For distant branes only massless states have a considerable contribution on the amplitude. Therefore we will restrict our attention to the long-range force. We know that the metric, antisymmetric tensor and dilaton states have zero winding numbers and zero momentum numbers, therefore from the general expression of the interaction amplitude (27), only the term with $N^{u_c} = 0$ (for all u_c) corresponds to this three massless states . We also must calculate the following limit

$$\Omega \equiv \lim_{q\to 0} \frac{1}{q} \prod_{n=1}^{\infty} \Big[(1-q^n)^2[det(1-Sq^n)]^{-1}\Big] \tag{33}$$

where $q = e^{-4t}$ and $S = S_1 S_2^T$, (note that we imposed $d = 26$). For a matrix A we have $detA = e^{Tr[lnA]}$ therefore

$$\Omega = \lim_{q\to 0} \frac{1}{q} \prod_{n=1}^{\infty} \Big[exp\Big(\frac{q^n}{n(1-q^n)}[Tr(S^n)-2]\Big)\Big] = \lim_{q\to 0} \frac{1}{q} + (TrS - 2) \tag{34}$$

The leading divergence is from the tachyon and we put away it, therefore we find

$$\begin{aligned} \mathcal{A}_0 &= \frac{T_{p_1}T_{p_2}}{4(2\pi)^{d_i}} \alpha' V_u \sqrt{det(1-\mathcal{F}_1)det(1-\mathcal{F}_2)}\ [Tr(S_1S_2^T)-2] \\ &\times \int_0^{\infty} dt \Big[\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{in}} e^{-\frac{1}{4\alpha' t}\sum_{i_n}(y_1^{i_n}-y_2^{i_n})^2} \prod_{i_c} \Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2}\Big)\Big] \end{aligned} \tag{35}$$

Note that the factor 2 in $[Tr(S_1S_2^T)-2]$ is purely due to the ghosts .

The interaction strength between two mixed branes can be read off from the above formula

$$T^2_{p_1,p_2} = \frac{T_{p_1}T_{p_2}}{24} \sqrt{det(1-\mathcal{F}_1)det(1-\mathcal{F}_2)}\ [Tr(S_1S_2^T)-2] \tag{36}$$

An overall factor $\frac{1}{24}$ is used to normalize the result .

Now consider two parallel m_p-branes with $\mathcal{F}_1 = \mathcal{F}_2 \equiv \mathcal{F}$ in non compact spacetime, therefore equation (35) gives

$$\mathcal{A}_0 = 6V_{p+1}[T_p^2 det(1-\mathcal{F})]G_{25-p}(Y^2) \tag{37}$$

where $G_D(Y^2)$ is the massless scalar Green's function in D dimensions and $Y^i = y_1^i - y_2^i$ is the separation of the m_p-branes .

Note that result (35) can also be obtained by projecting the boundary states onto the massless levels . Performing this projection on boundary state $\mid B_2\rangle$, the metric and the antisymmetric tensor components and dilaton part of this projection for $d = 26$ can be written as a state $\mid s_2\rangle$ which lives in $\mid B_2\rangle$ and has the form

$$\begin{aligned} \mid s_2\rangle &= \frac{T_{p_2}}{2}\sqrt{det(1-\mathcal{F}_2)}\ \delta^{(25-p_2)}(x^{i_2}-y_2^{i_2})(-\alpha^{\mu}_{-1}S^{(2)}_{\mu\nu}\tilde{\alpha}^{\nu}_{-1} \\ &+b_{-1}\tilde{c}_{-1}+c_{-1}\tilde{b}_{-1}) \mid Z\rangle \mid 0\rangle \prod_{\mu} \mid p^{\mu}_L = p^{\mu}_R = 0\rangle \end{aligned} \tag{38}$$

Vanishing of all left and right components of momentum says that this state has zero momentum and zero winding numbers . By inserting the closed string propagator between $\mid s_1\rangle$ (which lives in $\mid B_1\rangle$) and $\mid s_2\rangle$, again we obtain the result (35) .

5 Conclusion

We explicitly obtained boundary state for mixed boundary conditions on closed string and developed the corresponding formalism to extract the amplitude and the contribution of the massless states on it, in presence of non zero back-ground $B_{\mu\nu}$ and internal gauge fields .

The formalism was applied to both compact and non compact spacetime . In the space with compactification strong relation holds among internal windings and momenta restricting the total flux passing through the branes, or for a given flux restricting the windings and momenta of closed string. Only when the ratio of different total internal fluxes are fractional the winding modes of boundary states can have contribution to the interaction , since they can only emitted and absorbed with certain angles restricted by those fluxes .

The formalism can be extended to include fermionic degrees of freedom and hence can be applied to the superstring. This work is in progress.

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